Multi Scale Approximation of the Time-Dependent Boltzmann Equation

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Abstract *–A Multiple scale expansion is applied for the development of multi scale approximation functions to the P_1 approximation of the time dependent Boltzmann equation. Functions are developed for a homogenized cell with two effective groups of delayed neutrons and one energy group. The analytic functions are analyzed and compared with the solution of the diffusion equation and the solution for the P_1 equation gained by the quasi-static method. Differences in the time behavior are demonstrated on the basis of semi analytic functions.*

I. INTRODUCTION

The basis of transient simulations for nuclear reactor cores is the reliable calculation of the space, time, energy and angle dependent Boltzmann equation. The computation of exact solutions of this equation is very time consuming and for practical use approximated solutions are usually unavoidable.

A common scheme for developing higher order approximations involves expanding the angular dependence of the flux in spherical harmonics. The resulting time dependent P_1 equation cannot be separated in space and time. The aim of the work is creating analytic approximation functions for the representation of a multiplying system with delayed neutron production with its well-known stiff time behavior. These functions can be used either for the acceleration of 'exact' space time dependent solution methods via inversion by block partitioning of the main matrix or for developing effective approximation solutions with time steps independent of the neutron generation time.

Multiple scale expansion is the mathematical method used because it is well suited for the case of stiff system behavior where standard expansion methods can cause problems [1].

The multi scale functions give the possibility for a direct comparison of the solutions for the time dependent P_1 and the diffusion approximation and for solutions obtained by the quasi-static method.

II. MULTI SCALE FUNCTIONS

The usability, quality and efficiency of the method of multiple scale expansion for the development of approximation functions for time dependent approximations of the Boltzmann equation was demonstrated on the basis of the point kinetics equations. Excellent results in the representation of the stiff time behavior of a system with delayed neutron productions were shown [2, 3].

Following this basic demonstration, multi scale functions are developed for the P_1 equation for one homogenized cell and two effective groups of delayed neutrons. Space time dependent approximation functions of the kind

$$\begin{aligned}
&\phi(\xi,\tau_n) = \\
&\left[K_{x1}\sin(x_t\xi)+K_{x2}\cos(x_t\xi)\right]\sum_n D_{0n}e^{e_{1n}\tau_1+e_{2n}\tau_2}e^{e_0\tau_0} \\
&+ \\
&\left[K_{d1}\sin(x_t\xi)+K_{d2}\cos(x_t\xi)\right]\left(K_{B1}e^{e_3\tau_0}+K_{B2}e^{e_4\tau_0}\right) \\
&\qquad n = 1..2
\end{aligned} \tag{1}$$

are developed for the neutron flux, the neutron current and the precursor concentrations of both groups of delayed neutrons [2,4]. The functions are containing ξ dependent terms for the spatial distribution of the prompt flux and the delayed neutron precursor distribution. The exponential functions for the time dependent part are expressed in the multiple time scales τ_0 to τ_2. These time scales depend mainly on the neutron generation time and the decay constants of the delayed neutron.

A semi-analytical version of the multi scale functions is created to simplify the comparison of the results for the diffusion and the P_1 equation. The exact results for the diffusion equation can be calculated easily by using the separation of space and time. The arising functions for the space time dependent neutron flux for a positive perturbation (ρ = 0.3 \$) in a one dimensional slab system with no flux at the boundary the kinetic system parameters:
$\beta_1 = 4.277 \cdot 10^{-03}$, $\beta_2 = 3.223 \cdot 10^{-03}$, $\Lambda = 1.0 \cdot 10^{-06}$ s
$\lambda_1 = 2.926\ s^{-1}$, $\lambda_2 = 0.02613\ s^{-1}$ and the dimensionless constant for neutron transport $s_1 = 20$.

$$\phi_{dif}(\xi,\tau) = \cos(3.464\,\xi)\,(-0.4279\,e^{-0.00525\tau} - 1.441e^{-0.6062\,10^{-6}\tau} + 2.869\,e^{0.5403\,10^{-7}\tau}) \tag{2}$$

$$\phi_{P_1}(\xi,\tau) = \cos(3.464\,\xi)\,(-0.1103\,10^{-5}\,e^{-20.20\tau} - 0.4282\,e^{-0.00520\tau} - 1.441\,e^{-0.6062\,10^{-6}\tau} + 2.869\,e^{0.5403\,10^{-7}\tau}) \tag{3}$$

The equation for the neutron flux in the diffusion approximation (eq. (2)) can be developed easily by space-time separation. The result is the combination of the static cosine function and the exponential functions known from the point kinetics equations. This result would also arise for the solution of the P_1 equation by the quasi-static method. In contrast the results for the multi scale functions for the neutron flux in the P_1 equation is presented in eq. (3). There is an additional exponential function with negative exponent arising which is a result of the time derivative of the neutron current. Additionally the second exponent representing the prompt neutron production is changed slightly. These two changes influence the short time representation especially in the case of strong transients and die out as soon as the results are dominated by the delayed neutron production. The space time behavior of the difference in the neutron flux calculated following eq. (2) and eq. (3) is shown in Fig. 1.

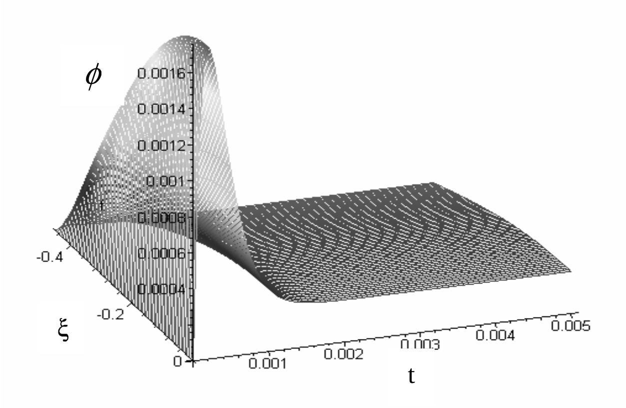

Fig. 1. Difference between the neutron flux calculated with the P_1 and the diffusion approximation.

The normalized spatial variable ξ and already the real time t are used not the dimensionless time τ anymore. The identical initial result for the diffusion and the P_1 equation (starting value of zero along the ξ axis) is a result of the identical static solution of these two equations. The positive peak in the beginning of the transient is a direct effect of the already described changes in the exponential functions for the P_1 equation. The amplitude and time duration of the peak is strongly system and perturbation dependent.

The reason for the difference is the time derivative term of the neutron current. This term is neglected in the development of the diffusion approximation and drops out during the development of the quasi-static solution too. This means only in the case of the recalculation of the distribution function in quasi-static calculations is this time dependent part of the neutron transport cared for.

III. CONCLUSIONS

The method of multiple scale expansion offers a new way for the development of very accurate and effective approximation functions for different approximations of the time dependent Boltzmann equation with delayed neutrons [2, 3, 4].

This method is used for the development of effective approximation functions for the time dependent P_1 equation with two groups of delayed neutrons. Analytic approximation functions are created for the neutron flux, the neutron current and the precursor concentrations for both groups of delayed neutrons for one homogenized cell. These functions give insight to the differences in time behavior between the P_1 and the diffusion equation.

Major differences are observed in the time behavior of the neutron current but also in minor scale in the short time behavior of the neutron flux while the influence on delayed production is negligible. A clear improvement of the time representation especially for short time periods arises compared to diffusion or quasi-static transport solutions.

As further step an effective approximation solution with an extended number of delayed neutron groups should be aimed at for the observation of longer time periods where the limitation on two groups of delayed neutrons could cause problems.

REFERENCES

1. J. KEVORKIAN, J. D. COLE: “Multiple Scale and Singular Perturbation Methods”, Appl. Math. Sc., Vol. 114, New York: Springer-Verlag 1996
2. B. R. MERK: „Eine mehrskalige Näherungslösung für die zeitabhängige Boltzmann-Transportgleichung“, FZKA 6963, Karlsruhe (2003).
3. B. R. MERK, D. G. CACUCI: “Multi Scale Approximation Solutions for the Time Dependent Boltzmann Equation: Part I - Point Kinetics Equations”, accepted by Nuc. Sc. and Eng.
4. B. R. MERK, D. G. CACUCI: “Multi Scale Approximation Solutions for the Time Dependent Boltzmann Equation: Part II – The P_1 and P_3 Equations”, accepted by Nuc. Sc. and Eng.